CHAPTER 25

Electromagnetic Chiral Materials

25.1 Introduction

A special class of electromagnetic (EM) materials referred to as *chiral materials* are emerging in engineering applications. A chiral medium is one whose electric and magnetic fields are *cross-coupled*. The characteristic aspect of such materials is the intrinsic *handedness* (right or left) present in their physical structure.

Optically active, natural materials exhibit mirror-asymmetric molecular structure(s) and have been originally known as chiral materials. Natural chiral structures include a diverse array of sugars, amino acids, DNA and certain mollusks as well as winding vegetations while the man-made versions encompass such objects as a helix, a Möbius strip, or an irregular tetrahedron. For example, a random suspension of metallic helical springs in a dielectric host constitutes a typical *electromagnetic chiral* medium.

As stated earlier, inherently, a chiral medium has left- or right-handedness in its microstructure with the result that a circularly polarized electromagnetic wave propagating through it would experience different phase velocities and/or absorption depending on it being left or right circularly polarized; and a rotation of the plane of polarization will be caused in a plane wave transmission through such a medium.

The concept of chiralic behavior of materials at suboptical (such as microwave, millimeter) wavelengths is of interest due to the feasibility of synthesizing such media as new types of electromagnetic composite materials. Considering a simple, isotropic, two-phase *achiralic medium*, the constitutive relations refer to $\boldsymbol{D} = \varepsilon_{eff}\boldsymbol{E}$ and $\boldsymbol{B} = \mu_{eff}\boldsymbol{H}$ where $\boldsymbol{D}$ and $\boldsymbol{B}$ are the electric and magnetic flux densities, respectively, and $\boldsymbol{E}$ and $\boldsymbol{H}$ depict the corresponding electric and magnetic field intensities. The macroscopic EM properties of materials, in general, are quantified by the effective permittivity (ε_{eff}) and the effective permeability (μ_{eff}) parameters. However, in the case of a chiralic mixture, the electric and magnetic fields are cross-coupled with the result the effective medium is modeled through the *cross-coupled constitutive relation(s)* written in the matrix form as:

$$\begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \varepsilon & \alpha \\ \alpha^* & \mu \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix} \tag{25.1}$$

where α is a complex number depicting the dimensionless *cross-coupling coefficient* and α^* is its conjugate. Explicitly $\alpha = (\chi - j\kappa)$ where χ is called the *Tellegen parameter* [1] measuring the nonreciprocal property of the medium. When $\chi = 0$ the medium is designated as a nonreciprocal chiral or *Pasteur medium* [2]; and the parameter κ decides the *degree of chirality*. When the chirality vanishes (with $\kappa = 0$), the medium represents a simple nonreciprocal achiralic material called the *Tellegen medium*. There are three well known versions of the cross-coupled constitutive relation given by Equation (25.1). These are known as *Post relation(s)*, *Condon-Tellegen relation(s)* and *Drude-Born-Fedorov relation(s)* [3]. Essentially, they all represent the electromagnetic constitutive relations of a chiralic medium but manifest in different algorithmic formats.

The chirality (right- or left-handedness) is a geometry-induced property of the medium which renders the medium to rotate the plane of polarization of a transmitted plane wave with respect to that of the incident plane wave. Classically, this property is referred to as the *optical rotary dispersion* (ORD). The cross-coupling between the field components in a chiral medium refers to the feasibility of an electric field *(**E**)* force inducing not only the electric displacement *(**D**)* or dielectric polarization but also a magnetic flux *(**B**)* or the magnetic polarization. Likewise, the magnetic field *(**H**)* impressed on a chiral medium would produce both magnetic and dielectric polarizations. The extent of such cross-coupled magnetoelectric effect is quantified by the chirality parameter κ.

Further, the handedness of the medium is represented by the quantity κ. When $\kappa > 0$ the medium is right-handed; and when $\kappa < 0$ the medium is left handed and the magnitude of κ (0 to ± 1) decides the amount of angular rotation that an incident plane wave would suffer in traversing such a medium. Also, the amount of rotation depends on the distance traveled in the medium; and this implies that the optical activity occurs not only at the surface but throughout the chiral medium. The constitutive parameters, namely, ε_{eff} and μ_{eff} are dependent only on the magnitude of the chirality factor (κ); that is, they remain the same for both right- or left-handedness of the medium.

25.2 State-of-the-Art Models of Chiralic Mixtures/Composite Materials

Although chiral materials have received attention only recently in electromagnetic applications, the concept of chirality and its role in a variety of fields like mathematics, chemistry, optics, and life sciences date back to the early 19th century. Electromagnetic chirality embraces both optical activity and *circular dichroism.* Optical activity refers to the rotation of the plane of polarization of optical waves by a medium while circular dichroism indicates a change in the polarization ellipticity of optical waves by a medium. The phenomenon of *optical activity* was first discovered by Arago in 1811 who found that a crystal of quartz rotates the plane of polarization of linearly polarized light. Pasteur [2] and Fresnel [4] also studied the phenomenon of optical activity.

Chirality and its effects attracted the attention of the electromagnetic community with the simple but illuminating microwave experiments of Lindman [5]. Regarding the analysis of wave interaction with the chiral media the work of Bassiri [6], Jaggard et al. [7], Silverman [8], and Lakhatia et al. [11,12] are more recent to note. Concerning the modeling of the effective parameters (permittivity and permeability) of a chiralic mixture the works of Sihvola and Lindell [9,10], Lakhatia et al. [11,12] are well known. A few well-researched applications of such chiral composites have been elaborated in [13-15]. Yet another possible approach in modeling the effective parameters of different types of chiralic mixtures usable at suboptical frequencies has been proposed by the author [16] and Subramaniam [17] as described below.

25.3 EM Chiralic Mixtures with Spherical Inclusions

The *logarithmic law* of mixtures (see Chapter 4) can be extended to a chiralic medium constituted by spherical chiralic inclusions dispersed in an achiralic host. Considering a two-phase, isotropic mixture formed by spherical chiralic inclusions of volume fraction θ and achiralic properties (ε_r, μ_r), embedded in an achiralic host medium of volume fraction equal to $(1-\theta)$ with electromagnetic property specified by (ε_2, μ_2), the corresponding cross-coupled values of the effective permittivity (ε_{eff}) and the effective permeability (μ_{eff}) of the mixture can be written as:

$$\varepsilon_{eff} = (C_1^{\theta} C_2^{(1-\theta)}) \{\varepsilon_{log} \mp \gamma (1/\eta_{log})\} \tag{25.2a}$$

$$\mu_{eff} = (C_3^{\theta} C_4^{(1-\theta)}) \{\mu_{log} \mp (1/\gamma)\, \eta_{log}\} \tag{25.2b}$$

where, $\varepsilon_{log} = \varepsilon_1^{\theta} \varepsilon_2^{(1-\theta)}$, $\mu_{log} = \mu_1^{\theta} \mu_2^{(1-\theta)}$, $\eta_{log} = (\varepsilon_{log}/\mu_{log})^{1/2}$ and γ is a cross-coupling coefficient. C_1, C_2, C_3, and C_4 are the weighting coefficients. These weighting coefficients are implicit parameters chosen to offer the attributes of logarithmic law of mixing to the effective permittivity and permeability properties. It may be noted that these coefficients weighted by the volume fraction exponents, namely, θ and $(1-\theta)$, are in the same analytical (logarithmic) form as described in Chapter 4. Further, the terms involving γ in Equation 25.2 represent the magnetoelectric crosscoupling due to the presence of chiralic inclusions.

Though, in general, ε_{eff} and μ_{eff} can be related (*via* a set of weighting functions) to the constituent values (namely, ε_r, μ_r, ε_2, μ_2, and γ) by any arbitrary function F_1 of Equation 4.8, the logarithmic format is currently chosen in conformity with the statistical/probabilistic attributes of the mixture as conceived by Lichtenecker and Rother [18].

The dimensional consistency in the above expressions is maintained *via* γ and $(1/\gamma)$ being used as appropriate. Further, (ε_1, μ_1) are the chiralic material parameters of the inclusions decided by their achiralic counterparts (ε_r, μ_r) and by the chirality factor ζ_1 of the inclusions. That is,

$$\varepsilon_1 = \varepsilon_r - (\eta_c\,\zeta_1)^{\eta\zeta_1+1} \tag{25.3a}$$

and

$$\mu_1 = \mu_r - (\eta_c\,\zeta_1)^{\eta\zeta_1-1} \tag{25.3b}$$

where η is the intrinsic impedance due to the achiralic parameters, namely, $(\mu_r/\varepsilon_r)^{1/2}$ and $\eta_c = \eta/[1 + (\eta\,\zeta_1)^2]^{1/2}$. In the above expressions it may be noted that in the case of the inclusions being achiralic (that is, $\zeta_1 \to 0$), $\varepsilon_1 = \varepsilon_r$ and $\mu_1 = \mu_r$.

The weighting coefficients of Equation 25.2, namely, C_1, C_2, C_3, and C_4, can be evaluated explicitly with the *geometrical mean constraint* that the effective characteristic impedance of the mixture, namely, $\eta_{eff} = (\mu_{eff}/\varepsilon_{eff})^{1/2}$, at the equivolume condition $(\theta = 0.5)$ would tend to the geometrical mean of its limiting values at $\theta = 0$ and 1; that is, at $\theta = 0.5$,

$$(\mu_{eff}/\varepsilon_{eff}) = \{(\mu_2/\varepsilon_2)^{1/2}\,(\mu_1/\varepsilon_1)^{1/2}\} \tag{25.4}$$

The foregoing geometrical mean constraint is the basis and the underlying principle of the logarithmic law expressed in the most general form by Equation 25.4. That is, in a truly stochastic mixture Lichtenecker [19] contended that the geometrical mean of the properties at the extremities of the volume fraction should correspond to the property at the mid-value of the volume fraction (that is, at $\theta = 0.5$). Hence, using this geometrical mean constraint specified above, the unknown coupling cofficient γ can be determined as 0, +1 or –1. The zero value applies when the inclusions are achiralic; and the +1 and –1 values refer to the right- or left-handedness of the inclusions and hence the mixture, respectively. Also the prefixed signs ($\mp$) or ($\pm$) for γ in Equation 25.2 account for the invariance of the effective parameters (ε_{eff} and μ_{eff}) with the handedness of the mixture.

Accordingly, the weighting coefficients C_1, C_2, C_3, and C_4 can be explicitly specified in terms of the material parameters as:

$$C_1 = \varepsilon_1/\{\varepsilon_1 \mp \gamma(\varepsilon_1/\mu_1)^{1/2}\}$$
$$C_2 = \varepsilon_2/\{\varepsilon_2 \mp \gamma(\varepsilon_2/\mu_2)^{1/2}\} \tag{25.5a}$$

and

$$C_3 = \mu_1/\{\mu_1 \pm (1/\gamma)(\mu_1/\varepsilon_1)^{1/2}\}$$
$$C_4 = \mu_2/\{\mu_2 \pm (1/\gamma)(\mu_2/\varepsilon_2)^{1/2}\} \tag{25.5b}$$

The effective chirality (ζ_{eff}) of the mixture is given by:

$$\zeta_{eff} = [(1+\zeta_1)^{\theta} - 1]\,\{(1/\eta_{eff})^2 - (1/\eta_{log})^2\}^{(1-\theta)/2} \tag{25.6}$$

The above expression for the effective chirality of the mixture is derived on the basis of the characteristic impedance relation:

$$\eta_{eff} = \eta_{log}/[1+(\eta_{log}\ \zeta_{eff})^2]^{1/2} \qquad (25.7)$$

where η_{eff} is the chiralic, and η_{log} the achiralic effective characteristic impedance of the mixture.

Inasmuch as the test medium represents a random mixture, the values of ε_{eff} and μ_{eff} should also be constrained by their corresponding upper and lower limits specified by the Wiener limits mentioned in Chapter 4. That is,

$$1/[\theta/\varepsilon_1 + (1-\theta)/\varepsilon_2] \leq \varepsilon_{eff} \leq \theta\varepsilon_1 + (1-\theta)/\varepsilon_2 \qquad (25.8a)$$

$$1/[\theta/\mu_1 + (1-\theta)/\mu_2] \leq \mu_{eff} \leq \theta\mu_1 + (1-\theta)/\mu_2 \qquad (25.8b)$$

The weighting coefficients as expressed in Equation 25.5 are the optimal values for a truly stochastic mixture inasmuch as they are derived on the basis of the geometrical mean constraint. If they are derived in any other possible way (such as on the basis of arithmetic mean constraint), the resulting values would not apply to a truely stochastic system.

The logarithmic law formulations of Equation 25.2 which refer to spherical chiralic inclusions can be extended to shaped chiralic inclusions as well *via* Fricke's formula by incorporating the explicit dependency of the results on the aspect ratio or eccentricity of the inclusions.

25.4 Chiralic Composites with Shaped Inclusions

In the previous section, analytical descriptions for the effective parameters of a simple chiralic mixture randomly dispersed with spherical chiralic inclusions were indicated. The problem of shaped chiralic inclusions randomly dispersed in an achiralic host could also be addressed on the basis of mixture theory. The effective values of the dielectric permittivity and magnetic permeability of such a composite medium are derived by modifying Fricke's formula on the basis of the logarithmic law of mixing. The resulting expressions are thus *ad hoc* extensions of the approaches due to the well-known Fricke's and logarithmic law formulations.

In general, as discussed in Chapter 4, the particulate inclusions are referred to as shaped if two or more of the lateral dimensions are significantly different as in the case of ellipsoids, prolate/oblate spheroids, needles, and disks. For a spheroidal geometry with semi-axes a, b, and c and taking $b = c$, the aspect ratio is equal to (a/b). When this aspect ratio is of significant value (either large or small compared to unity) the corresponding eccentricity (e) would play a significant role in the polarization of the particles when the mixture is submitted to an external field; and the depolarization arising from the relative disposition of the particles due to the random nature of the particle dispersion (and/or orientation) in the mixture would become another effective stochastic parameter to be duly considered. The works of Wiener, Fricke, Sillars, Lewin, Hamon, and Boned and Peyrelasse are the well-known endeavors directed towards the elucidation of the dielectric properties of simple achiralic mixtures with shaped inclusions as discussed in Chapter 4.

Considering a chiralic mixture composed of an achiralic host dispersed randomly with shaped chiralic inclusions (such as helices) which have an inherent shape factor associated with them, no well-known formulations are presently available. Hence, in the following section analytical descriptions for such a mixture are developed and some theoretical results and experimental data are presented.

25.5 Effective Parameters of Chiralic Mixtures with Shaped Inclusions

For a spheroidal geometry of the inclusions with semi-axes a, b and c and taking $b = c$, the aspect ratio is equal to (a/b); and in Fricke's formulation as well as in [20], a shape factor denoted by x was chosen to represent the dependency of the permittivity on the aspect ratio. The corresponding factor for the permeability is taken as y currently. Using the method of [20], the modified shape factor(s) for a chiral mixture constituted by an achiralic host and shaped chiralic inclusions can be deduced as follows.

In terms of Fricke's formulation [20], the effective permittivity (ε_{eff}) is given by:

$$\varepsilon_{eff} = \{\varepsilon_1\varepsilon_2 (1 + x\theta) + \varepsilon_2^2 x(1 - \theta)\}/\{\varepsilon_1(1 - \theta) + \varepsilon_2(x + \theta)\} \tag{25.9}$$

Analogously the effective permeability (μ_{eff}) of the mixture is given by:

$$\mu_{eff} = \{\mu_1\mu_2 (1 + y\theta) + \mu_2^2 y(1 - \theta)\}/\{\mu_1(1 - \theta) + \mu_2(y + \theta)\} \tag{25.10}$$

Setting these expressions identically equal to the respective relations of Equation 25.10, the shape factors x and y are obtained as:

$$x = M\,N_x/D_x \tag{25.11a}$$

where $N_x = \{C_1^{\theta}C_2^{(1-\theta)}[\varepsilon_1(1 - \theta) + \varepsilon_2\theta](\varepsilon_{log} \mp \gamma(1/\eta_{log})) - \varepsilon_1\varepsilon_2\}$
and $D_x = \{\varepsilon_2[\varepsilon_2(1 - \theta) + \varepsilon_1\theta - C_1^{\theta}C_2^{(1-\theta)}\{\varepsilon_{log} \mp \gamma(1/\eta_{log})\}]\}$

$$y = N\,N_y/D_y \tag{25.11b}$$

where $N_y = \{C_3^{\theta}C_4^{(1-\theta)}[\mu_1(1 - \theta) + \mu_2\theta](\mu_{log} \pm \gamma(1/\eta_{log})) - \mu_1\mu_2\}$
and $D_y = \{\mu_2[\mu_2(1 - \theta) + \mu_1\theta - C_3^{\theta}C_4^{(1-\theta)}\{\mu_{log} \pm \gamma(1/\eta_{log})\}]\}$

The parameters M and N in Equation (25.11) are factors dependent on the (a/b) ratio of the spheroidal inclusions. For an oblate spheroid $(a > b)$ or disk-like $(a >> b)$ inclusions, $M = 2/(m - 1)$ if $\varepsilon_1 \geq \varepsilon_2$ or $(m - 1)/2$ if $\varepsilon_1 \leq \varepsilon_2$. Likewise $N = 2/(m - 1)$ if $\mu_1 \geq \mu_2$ or $(m - 1)/2$ if $\mu_1 \leq \mu_2$. Here m refers to the depolarization factor given by [23]:

$$m = 1/\{1/(1 - f^2) - [f \arccos(f)/(1 - f^2)^{3/2}]\} \tag{25.12}$$

where $f = (b/a) < 1$ and the eccentricity e (of the oblate spheroid) is equal to $1 - (b/a) = (1 - f)$.

For prolate spheroidal $(a < b)$ or needle-like $(a << b)$ inclusions, $M = 2/(m - 1)$ if $\varepsilon_1 \leq \varepsilon_2$; or $(m - 1)/2$ if $\varepsilon_1 \geq \varepsilon_2$. Likewise $N = 2/(m - 1)$ if $\mu_1 \leq \mu_2$; or $(m - 1)/2$ if $\mu_1 \geq \mu_2$. In this case m is given by [23]:

$$m = 1/\{1/(g^2 - 1) - [g\ln(g + (g^2 - 1)^{1/2})/(g^2 - 1)^{3/2}]\} \tag{25.13}$$

where $g = b/a > 1$; and the eccentricity e (of the prolate spheroid) is equal to $1 - (a/b) = 1 - (1/g)$.

Hence, the effective permittivity (ε_{eff}) and effective permeability (μ_{eff}) of the test mixture can be calculated from Equations 25.9 and 25.10 with the values of x and y of Equation 25.11. The effective chirality (ζ_{eff}) of the mixture can then be evaluated from Equation 25.6.

Sample computations on the above algorithms were performed with the following data. A test mixture is presumed to consist of an achiralic background with shaped chiralic inclusions of volume fraction $\theta = 0.4$. Two sets of hypothetical ingredients were considered, namely, ($\varepsilon_r = 78.3$, $\mu_r = 1000$, $\varepsilon_2 = 2$, and $\mu_2 = 55$) and ($\varepsilon_2 = 78.3$, $\mu_2 = 1000$, $\varepsilon_r = 2$, and $\mu_r = 55$). In each case, an arbitrary chirality factor of $\zeta_I = 0.0001$ was presumed. That is, the chosen value of ζ_I represents the degree of chirality decided by the distinct shape of the inclusions (such as helical); and the magnitude of ζ_I (taken here as 0.0001) would alter the achiralic parameters (ε_r, μ_r) of the inclusions to the corresponding chiralic values, namely, (ε_I, μ_I) *via* the relations given by Equation 25.11. Thus ζ_I controls the degree of chirality and can be designed by appropriate geometry of the inclusions. Inasmuch as the inclusions are chiralic, the resulting mixture would also exhibit chiralic characteristics with an effective chirality factor given by Equation 25.6.

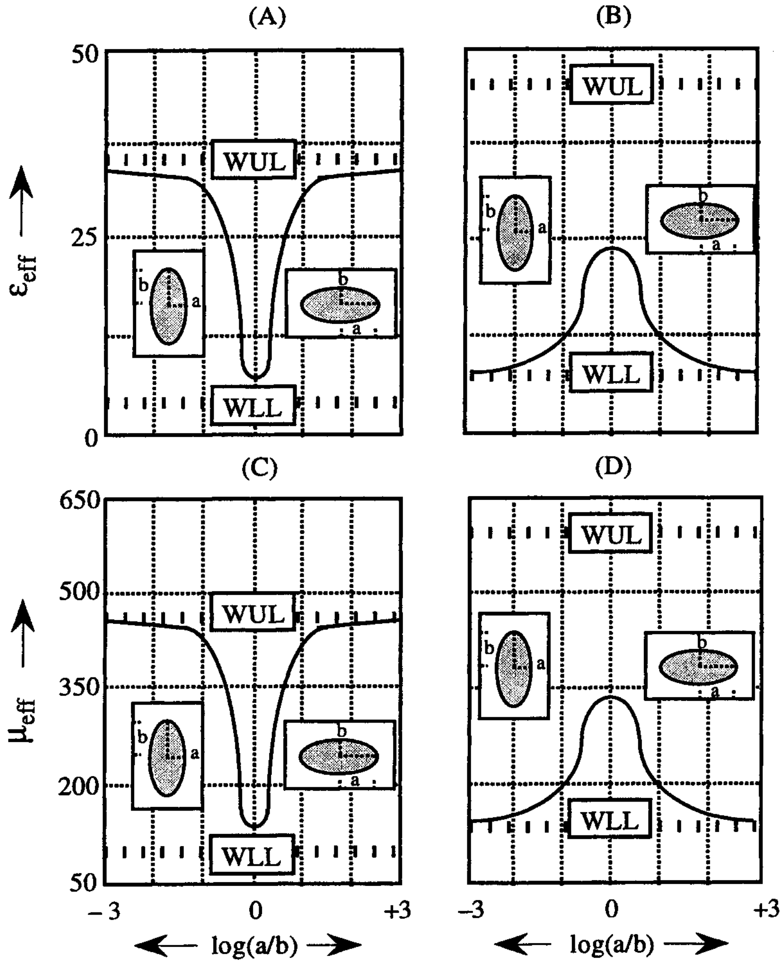

Figure 25.1 Effective values of permittivity (ε_{eff}) and permeability (μ_{eff}) of a chiralic mixture *versus* aspect ratio (a/b) of the inclusions of volume fraction θ.
(θ = 0.4; WUL: Wiener's upper limit; WLL: Wiener's lower limit. For A&C: $\zeta = +1$, $\zeta = 0.0001$, $\varepsilon_r = 78.3$, $\mu_r = 1000$, $\varepsilon_2 = 2$, $\mu_2 = 55$; For B&D: $\zeta = +1$, $\zeta = 0.0001$, $\varepsilon_2 = 78.3$, $\mu_2 = 1000$, $\varepsilon_r = 2$, $\mu_r = 55$.)

In the foregoing theoretical considerations, the chiralic particulates dispersed in the host medium can be "stretched" or "compressed" so that each particle would assume an axially asymmetric or "shaped" chiralic geometry. In this case the particulate eccentricity *(e)* or the aspect ratio *(a/b)* should also be considered. Hence, for different values of *(a/b)*, the computed results on the two hypothetical samples considered are presented in Figure 25.1.

The inferences from the results pertinent to Figure 25.1 are:

- The effective dielectric permittivity and magnetic permeability of a random chiralic mixture are functions of the shape factor of the inclusions as in the case of achiralic mixtures.
- The material parameters given by Equations 25.9 and 25.10 reduce to that of a mixture with spherical inclusions as in Equation 25.10 when a/b = 1.
- Also, the formulations of Equations 25.9 and 25.10 are in a closed form.
- Use of the logarithmic law of mixing confirms the proportionality postulation applicable to a statistical mixture.
- The expressions for ε_{eff} and μ_{eff} satisfy conditions at the extreme limits of $\theta = 0$ or 1.
- The results indicated are bounded by the Wiener limits (see Chapter 4).
- Last, these formulations based on the logarithmic law of mixing refer only to randomly dispersed spheroidal inclusions (disordered systems) in an achiralic host and do not apply when the shaped inclusions are aligned/oriented specific to the electric field direction. The algorithms, however, can be modified to suit such orderly disposed inclusions and are detailed in Section 25.8.

From the results presented, it could be evinced that, for a given set of constituent parameters, ε_{eff} and μ_{eff} vary significantly with respect to the particulate eccentricity and eventually approach their Wiener bounds at the limiting values of the aspect ratio corresponding to the particulate shape becoming disk-like or needle-like as depicted in Figure 25.1.

25.6 Practical Considerations: An Experimental Study

To illustrate the practical aspects of using the algorithms of the previous section in synthesizing a chiral electromagnetic composite for applications at quasioptical frequencies, the following experimental study as reported in [17] can be considered.

A square slab of 11.85 x 12.5 x 2.72 cm of a test composite was fabricated with a host medium of paraffin wax ($\varepsilon_2 = 2.35$, $\mu_2 = 1$) embedded with a large number (approximately 120) of miniature right-handed helical springs made of a high permeability metal alloy with $\mu_r = 30{,}000$. The radius of each helix is 0.15 cm and the pitch is 0.25 cm and it contains 3.5 turns. The metallic volume is about 2% of the cylinder of radius 0.15 cm and height 0.9 cm. Considering the total volume of the slab, this cylindrical volume constitutes a volume fraction of 0.2. At the test wavelength of about 3 cm, the chiral mixture so fabricated is fairly homogeneous by virtue of the size of the helices being small in comparision to the operating wavelengths. However, the chiral inclusions should not be too small lest they become "invisible" to the propagating wave. Further, the helices were disposed randomly in the wax medium so as to emulate a truly statistical mixture and ensuring isotropic performance.

The metal concentration of the right-handed helices, namely, 2% refers to the corresponding cylindrical volume fraction occupied by the helices of the order of 20%. Hence, notwithstanding the actual volume fraction of the metal appearing low, the apparent cylindrical volume enclosed by the springs is quite high. Considering the dimensions of the springs, the ratio of the length of the cylindrical volume enclosed to the radius of the helix specifies the aspect ratio *(b/a)* of the inclusions. In the present case, it is equal to about 6.

The influence of the effective electromagnetic properties of the test sample on microwave transmission was studied at X-band frequencies (8 to 10 GHz). For this purpose, a microwave transmitter-receiver arrangement was used with the test slab irradiated by a focused beam of microwaves emerging from a microwave horn.

For a given frequency setting with the transmitter horn launching a vertically polarized transmitted (E_{yI} component only) beam, the receiver (also set to receive the vertically polarized wave) was calibrated and normalized so that the detected output corresponds to 0 dB. This refers to a total free-space transmission with a coefficient of 1. As a next step, the test chiralic slab was introduced between the transmitter and the receiver horns and the following resulting effects at the receiver were measured:

(1) The power transmission coefficient due to vertically polarized transmitted and received E_{yT} waves (that is, $|E_{yT}/E_{yI}|^2$)

(2) The power transmission coefficient due to vertically polarized transmitted wave and the cross-polarized (orthogonal) received E_{zT} wave (that is, $|E_{zT}/E_{yI}|^2$)

These measurements were performed at different spot frequencies over 8 to 10 GHz range. Relevant results are presented in Figures (25.2 and 25.3). To compare the experimental data with the theoretical calculations, the values of ε_{eff} and μ_{eff} as given by Equations 25.9 and 25.10 were computed with the following data relevant to the test slab: $\varepsilon_2 = 2.35$, $\mu_2 = 1$, $\mu_r = 30000$, $\theta = 0.2$ and $a/b = 1/6$. Inasmuch as the inclusion is made of metal and its corresponding value of ε_r is not definable, the use of the logarithmic law of mixing (when the inclusions are metallic) is usually questioned. However, the author [21] had developed an exclusive method to obviate this difficulty by extending the logarithmic law to complex dielectric susceptibility which accounts for such metallic inclusions. Hence, following the method given in [21], for the metallic inclusions of conductivity σ_I, the expression for ε_{log}, ε_r and ε_I of Equation 25.9 can be written as:

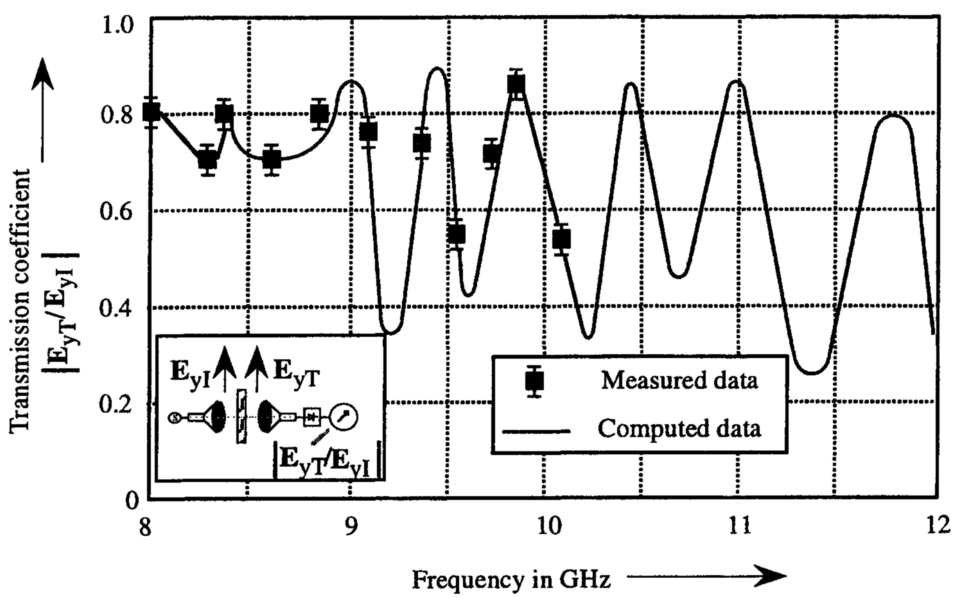

Figure 25.2 Computed and measured data on transmission coefficient *versus* frequency for a normally incident E-polarized beam wave on a chiralic composite slab.

$$\varepsilon_r \rightarrow (\sigma_I/\omega\varepsilon_0)$$
$$\varepsilon_I \rightarrow \Omega = [(\sigma_I/\omega\varepsilon_0) - (\eta_c\zeta_I)^{\eta\zeta_I} + 1] \qquad (25.14)$$

and

$$\varepsilon_{log} \rightarrow \Omega^{\theta}\{(\varepsilon_2 - 1)^{1-\theta}\cos(\pi\theta/2)\} + 1$$

where $\omega = 2\pi x$ frequency and ε_0 = free-space permittivity.

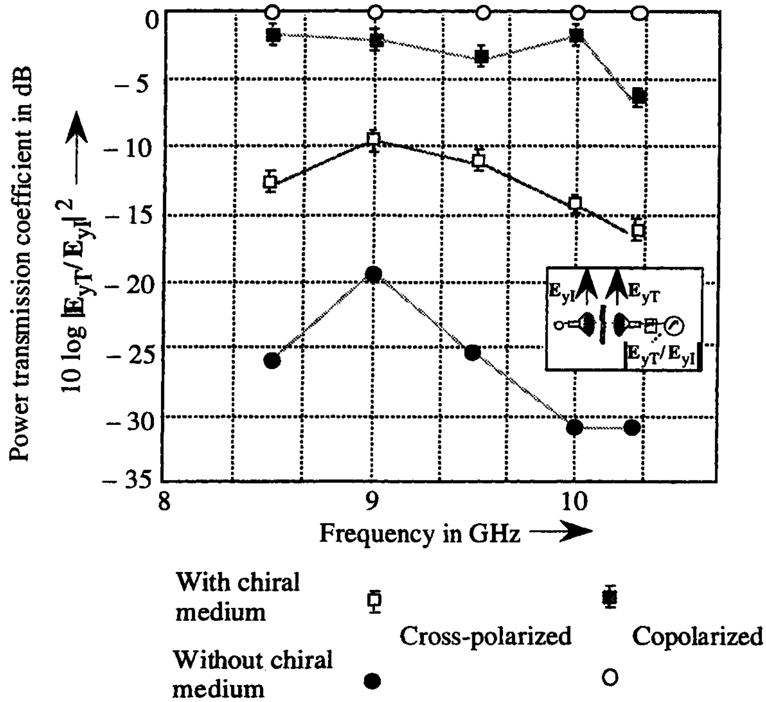

Figure 25.3 Data on co- and cross-polarized power transmission coefficients *versus* frequency measured with a E-polarized beam wave normally incident on a chiralic composite slab.

It may be noted that the above expressions are frequency dependent. Therefore, the corresponding results of Equations 25.9 and 25.10 would also be frequency dependent. For the test sample under discussion the metallic inclusions have $\sigma_I = 1.8 \times 10^6$ S/m. Hence, to pursue the calculations with Equations 25.9 and 25.10, the only unknown left is the chirality factor ζ_I of the inclusions.

For a given set of values, namely, (ε_{eff}, μ_{eff} and η_{eff}), the corresponding transmission coefficient (for normal incidence) at the metal-backed test composite is given by:

$$|T| = \{|\Gamma - \Gamma exp[-j4\pi(\varepsilon_{eff}\mu_{eff})^{1/2} d/\lambda_0]|\}/\{|1 - \Gamma^2 \, exp[-j4\pi(\varepsilon_{eff}\mu_{eff})^{1/2} d/\lambda_0]|\} \quad (25.15)$$

where $j = \sqrt{-1}$, d is the thickness of the slab, and λ_0 is the free-space wavelength.

Considering a test frequency (in the range 8-12 GHz), the computed data on ε_{eff}, μ_{eff}, and η_{eff} of the test sample are presented in Figures 25.4–25.6. These results show that regardless of the value of ζ_I (in the range $0 < \zeta_I < 1$), the calculated values of ε_{eff}, μ_{eff}, and η_{eff} remain almost unchanged.

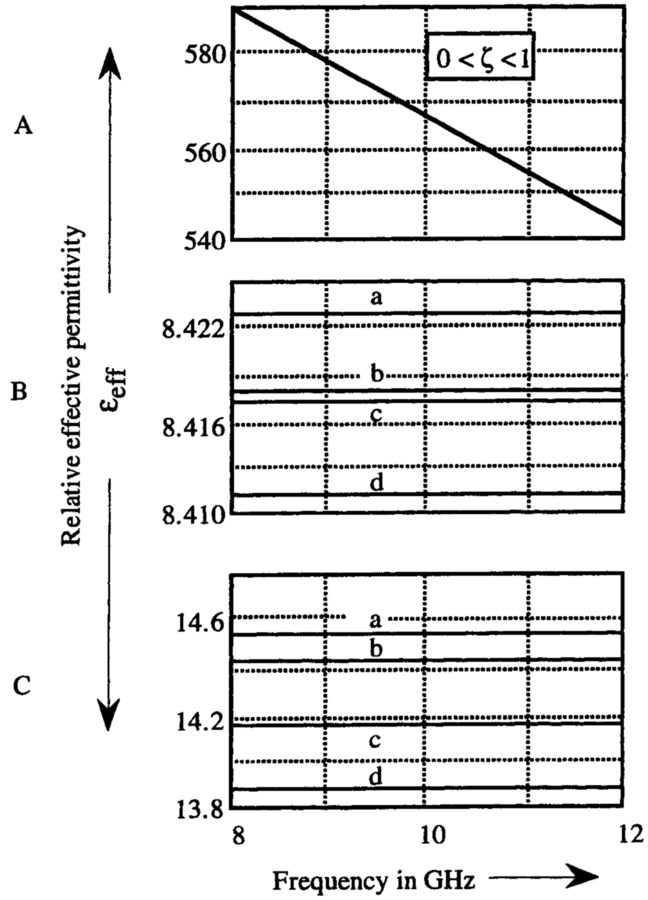

Figure 25.4 Computed data on the relative effective permittivity *versus* frequency pertinent to chiralic composite media with $0 < \zeta_1 < 1$.

Data on the test samples: A: $\varepsilon_2 = 2.35$, $\mu_2 = 1$, $\sigma_1 = 1.8 \times 10^6$ siemen/meter, $\mu_r = 30000$.
B: Hypothetical sample 1: $\varepsilon_2 = 2$, $\mu_2 = 55$, $\varepsilon_2 = 78.3$, $\mu_r = 1000$.
C: Hypothetical sample 2: $\varepsilon_r = 2$, $\mu_r = 55$, $\varepsilon_2 = 78.3$, $\mu_2 = 1000$.

For the hypothetical samples on the other hand with nonmetallic composition *(*ε_r = *78.3,* μ_r = *1000,* ε_2 = *2,* and μ_2 = *55)* and *(*ε_2 = *78.3,* μ_2 = *1000,* ε_r = *2,* and μ_r = *55)* as considered in the previous section, the corresponding values of ε_{eff}, μ_{eff}, and η_{eff} (as presented in Figures 25.4 to 25.6 for a volume fraction $\theta = 0.2$ and $a/b = 1/6$ over the frequency range of 8-12 GHz) show that for different values of ζ_l (in the range of 0 to 1) the effective parameters of these hypothetical samples may vary significantly.

As regards the hypothetical samples and the test sample, the major difference in their electromagnetic constitutive characteristics is that in the test sample the fabricated inclusions are conductors contributing a susceptance term $(\sigma_l/\omega\varepsilon_0)$ of excessive magnitude whereas this is replaced by the ε_l term in the hypothetical samples which is relatively of very small value. Further, unlike in the test sample, ε_l and μ_l are assumed to be frequency independent in the hypothetical samples.

Hence, it can be surmised that the overwhelming dielectric susceptance contribution due to the conductor inclusions make the effective parameters of the mixture dependent on the frequency but insignificantly dependent on the ζ_l values. In contrast, the low dielectric susceptance due to the small values of ε_l (such as those for dielectric inclusions considered in the hypothetical samples) render the effective parameters dependent on ζ_l, but independent of the frequency.

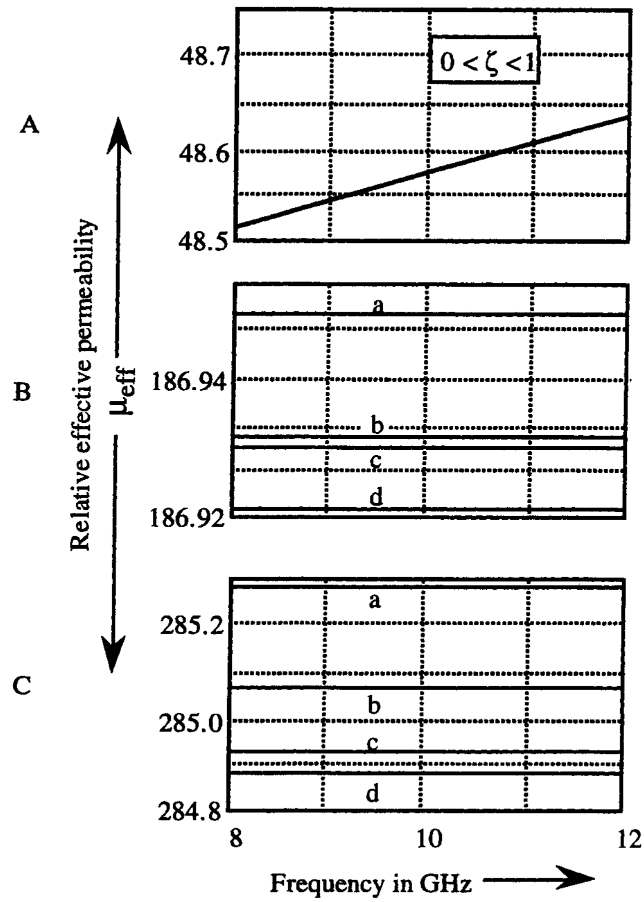

Figure 25.5 Computed data on the relative effective permeability *versus* frequency pertinent to chiralic composite media with $0 < \zeta_1 < 1$.

Data on the test samples: A: $\varepsilon_2 = 2.35$, $\mu_2 = 1$, $\sigma_1 = 1.8 \times 10^6$ siemen/meter, $\mu_r = 30000$.

B: Hypothetical sample 1: $\varepsilon_2 = 2$, $\mu_2 = 55$, $\varepsilon_2 = 78.3$, $\mu_r = 1000$.

C: Hypothetical sample 2: $\varepsilon_r = 2$, $\mu_r = 55$, $\varepsilon_2 = 78.3$, $\mu_2 = 1000$.

Now referring to Figure 25.2, the calculated values as per Equation (25.15) of the transmission coefficient corresponding to the co-polarized components of the incident (vertically polarized) beam wave, vary with frequency exhibiting resonant windows. This is confirmed by the measured values.

Further, the presence of the test slab in the transmission path causes a rotation of the plane of polarization of the incident beam wave. This is evinced from the measured cross-polarized (orthogonal) field component at the receiver. The ratio of the co-polarized and cross-polarized field components at the receiver therefore refers to the ellipticity of polarization caused by the chiralic property of the test sample. Hence, it should implicitly depict the effective chirality of the test medium, namely, ζ_{eff}. From ζ_{eff} using Equation 25.14 the corresponding values of ζ_l can be determined. Relevant results are also presented in Table 25.1. It may be noted that both ζ_{eff} as well as ζ_l are almost invariant with frequency suggesting that they are more dependent on the geometry of the included chirals rather than on the external electromagnetic field forces.

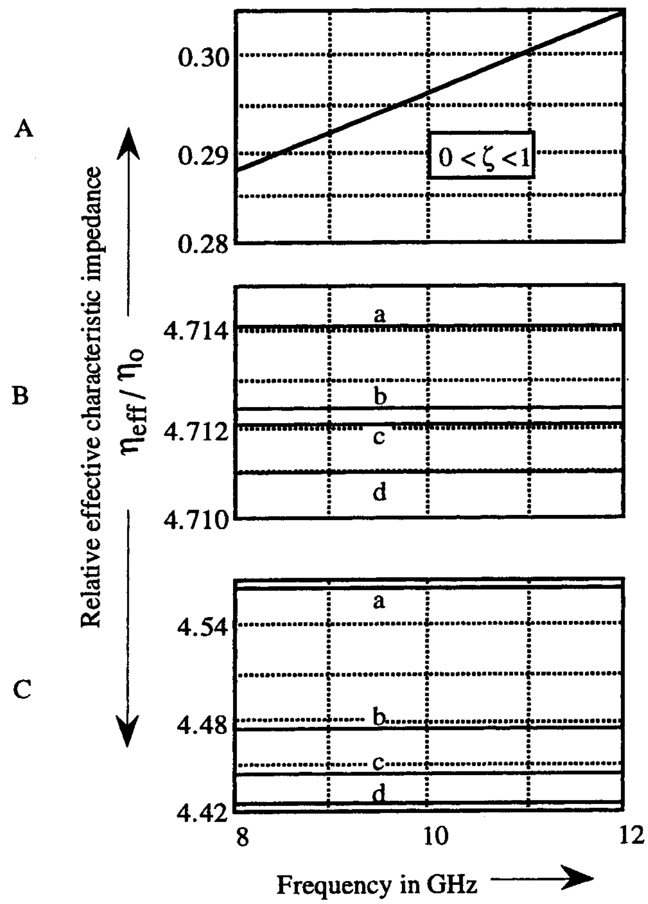

Figure 25.6 Computed data on the relative effective characteristic impedance *versus* frequency pertinent to chiralic composite media with $0 < \zeta_1 < 1$ ($\eta_o = 120\pi$ ohms).
Data on the test samples: A: $\varepsilon_2 = 2.35$, $\mu_2 = 1$, $\sigma_1 = 1.8 \times 10^6$ siemen/meter, $\mu_r = 30000$.
B: Hypothetical sample 1: $\varepsilon_2 = 2$, $\mu_2 = 55$, $\varepsilon_2 = 78.3$, $\mu_r = 1000$.
C: Hypothetical sample 2: $\varepsilon_r = 2$, $\mu_r = 55$, $\varepsilon_2 = 78.3$, $\mu_2 = 1000$.

25.7 Discussions on the Theoretical Considerations

On the basis of the theoretical aspects presented and the experimental results furnished in the previous sections the following can be inferred:

1. Although the formulations indicated are extrapolations of the logarithmic law of mixing as applied to dielectric permittivity, its applicability to practical systems is evident from the theoretical and experimental results furnished. Close correlation between the theoretically evaluated transmission coefficients (in terms of ε_{eff} and μ_{eff}) and the experimentally determined values at different frequencies (Figure 25.2) validates the algorithms of Equations 25.9 and 25.10.
2. A chiralic mixture constituted by a random dispersion of conducting chirals in an achiralic host offers electromagnetic absorption characteristics as is evident from the measured transmission coefficient results depicted in Figure (25.2).

3. Such electromagnetic absorption is frequency dependent as could be seen in Figure (25.2). This frequency dependency arises from:
   (a) The loss tangent of the host dielectric. (This, however, being very small for paraffin wax, could be neglected).
   (b) The dissipative loss is due to conducting inclusions as dictated by Equation 25.14. This is rather a more predominant factor than the loss-tangent effects of the host medium.
4. Both theoretical calculations and experimental data (Figures 25.2 and 25.3) indicate that the absorption characteristics may exhibit resonances. Although this corresponds to limited bandwidth of operation, these resonances can be quenched with high concentration of inclusions as indicated in [22].
5. Further, the dispositions of inclusions may also affect the bandwidth performance. Especially, the inclusion-to-inclusion contiguity will decide the relaxation and hence the effective absorption process.
6. Such contacts between the inclusions would also affect the chirality of the composite due to induced surface currents on the helices [22].
7. The indicated study has addressed implicit definitions for the effective chirality (ζ_{eff}) of the mixture and for the chirality of the inclusions (ζ_I) in terms of experimental parameters as depicted in Table 25.1. In the relevant work as well as in the other existing studies [9,10], there is no method of knowing the value of the intrinsic chirality parameter (ζ_I) of the inclusions on *a priori* basis, except that it is controlled by the size and geometry of the inclusions. However, as proposed above a strategy to measure the effective chirality of the mixture-medium (namely, ζ_{eff}) is feasible, and thereby the value of ζ_I on *a posteriori* basis can be deduced (Table 25.1).

Table 25.1 Estimation of the Intrinsic Chirality of the Inclusions

Frequency in GHz	Measured Cross-polarization power C_p in dB relative to free-space transmission	Proposed measure of effective chirality $\varepsilon_{eff} = \{\text{antilog}[-C_p/10]\}^{-1/2}$	ε_1 as determined from ε_{eff} *via* Equation (25.6)
8.5	10	0.3162	0.9860
9.0	10	0.3162	0.9930
9.5	12	0.2512	0.7500
10.0	11	0.2818	0.8700
10.2	10	0.3162	0.9990

8. The results further indicate that ε_{eff} and μ_{eff} are frequency dependent (Figures 25.4 and 25.5) mainly due to the complex permittivity of the inclusions. The effective chirality of the overall mixture as given by Equation 25.14 is, however, only slightly frequency dependent as decided by the transmission coefficient(s) involving the electrical thickness (d/λ_0) of the test sample. This is confirmed by a minor variation in the measured chirality-dependent cross-polarized component *versus* frequency in Figure 25.3.

To conclude, the approach pursued in [17] refers to the extension of the well-known logarithmic law of mixing on an *ad hoc* basis to determine the effective electromagnetic constitutive parameters of chiralic mixtures.

The theoretical results so obtained are supplemented by some experimental results to portray the design feasibilities of realizing chiral composites at microwave frequencies. A variety of applications of such composites have been considered in practice [13–15]. To name a few, EMI shields, radar absorbing materials, polarizing lenses, etc. can be designed to match certain specific characteristics with the type of chiralic mixtures discussed.

The formulations indicated in this chapter are useful in synthesizing chiralic composites with chiralic inclusions of size comparable to the wavelengths of operation. If the size of the inclusions is too small (in comparison to the wavelength), it will render the medium achiralic. Then the present formulations will refer to an achiralic mixture medium with $\zeta_I = 0$.

Last, the following should be noted concerning the theoretical considerations of this chapter which are based on combining Fricke's formula and the logarithmic law of mixing as applied to a chiralic composite. Unlike Maxwell-Garnett's formulation, Fricke's formula (see Chapters 4–7) is devoid of dilute-inclusion approximation; and the shape factor is controlled in the analysis presented here by the logarithmic law, permitting the mixture to be viewed as a stochastic entity with macroscopic random attributes to its constitutive parameters. Thus, although not derived on the basis of the first principles, the algorithms of this chapter are in line with the electrostatic aspects of Fricke's formula and the probabilistic characteristics of a statistical mixture.

25.8 Orderly-Textured Chiralic Mixture Media

In Chapter 4 (Section 4.5), the case of an orderly-textured simple (achiralic) dielectric mixture was discussed and relevant formulations were presented. Now, relevant to the discussion on chiralic mixtures a new class of chiralic mixtures called *orderly-textured chiralic mixtures* are considered. Such mixtures can be constructed by an ordered arrangement of shaped, chiralic inclusions in an achiralic host. The effective electromagnetic properties of such a mixture would be considerably different from those of both simple orderly-textured media as well as random chiralic media discussed earlier.

In order to describe the effective electromagnetic properties (permittivity and permeability) of an orderly-textured chiralic mixture, the formulations indicated in Chapter 4 can be utilized and a strategy similar to that of the foregoing sections can be adopted. In the previous sections the logarithmic law was extended in a modified form to describe the effective properties of a chiralic mixture comprised of an achiralic host and randomly dispersed with shaped chiralic inclusions. Accordingly, the formulations for the effective permittivity and the effective permeability of such a mixture are as given by Equations (25.9) and 25.10.

Now utilizing the above formulations and following a weighting strategy similar to that adopted for the orderly-textured simple (achiralic) mixture, analytical formulations for the effective properties of an orderly-textured chiralic mixture can be developed. That is, corresponding to the Equations of 4.23 and 4.24, the formulations for the effective permittivity (ε_{eff}') and the effective permeability (μ_{eff}') for an orderly-textured chiralic mixture can be written as follows:

$$\varepsilon_{eff}' = \varepsilon_{eff}(\varepsilon_U/\varepsilon_{eff})^r \tag{25.16a}$$

$$\mu_{eff}' = \mu_{eff}(\mu_L/\mu_{eff})^r \tag{25.16b}$$

and

$$\varepsilon_{eff}' = \varepsilon_{eff}(\varepsilon_L/\varepsilon_{eff})^n \tag{25.17a}$$

$$\mu_{eff}' = \mu_{eff}(\mu_U/\mu_{eff})^n \tag{25.17b}$$

All the notations and other conditions or statements as regards to the orientation specified with respect to the electric field direction remain the same as those for a simple orderly-textured dielectric mixture discussed in Chapter 4.

The above formulations (Equations 25.16 and 25.17) are concerned only with the interaction of an orderly-textured chiralic mixture with the direction of an external applied electric field. However, it should be noted that inasmuch as an externally applied magnetic field can also evoke a response in a chiralic medium, the corresponding formulations for ε_{eff}' and μ_{eff}' would be different from Equations 25.16 and 25.17.

It should be observed that even though the effective permittivity (ε_{eff}) alone was discussed for the case of orderly-textured achiralic mixtures presented in Chapter 4, formulations for both ε_{eff}' and μ_{eff}' are to be considered in the case of such chiralic mixtures. This is due to the fact that in a chiralic mixture (unlike an achiralic mixture) the electric and magnetic fields are cross-coupled.

In summary, proceeding from the simple logarithmic law of mixing weighted on the basis of Langevin's function, the effective permittivity of a simple, orderly-textured dielectric mixture can be elucidated. Hence, the effective permittivity and permeability of an orderly-textured, chiralic mixture can also evaluated with the aid of the results presented in the previous sections.

25.9 Sample Results on Orderly-Textured Chiralic Mixtures

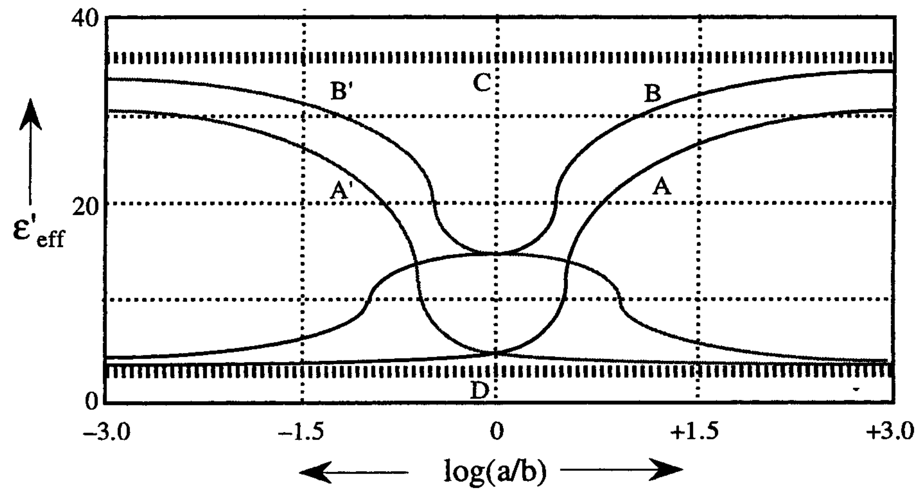

Figure 25.7 Effective permittivity (ε_{eff}') of an orderly-textured chiralic mixture *versus* the (a/b) ratio of the inclusions of volume fraction, $\theta = 0.4$.
(Data: $\varepsilon_r = 78.3$; $\mu_r = 1000$; $\varepsilon_2 = 2$; $\mu_2 = 55$; $\gamma = +1$; $\zeta = 0.0001$.)
AA': Sihvola and Lindell's [10] formulations for perpendicular orientation (A) or parallel (A') to the applied electric field (E); BB': Corresponding results with the formulations of Equations 25.16 and 25.17; C: Wiener's upper limit; D: Wiener's lower limit.

The results corresponding to an orderly-textured chiralic mixture as per Equations 25.16 and 25.17 are compared with those due to Sihvola and Lindell [10] in Figure 25.7.

- It is observed from Figure 25.8 that for prolate spheroidal inclusions the value of ε_{eff}' tends towards Wiener's upper limit and μ_{eff}' to Wiener's lower limit in the limiting case of all inclusions being aligned parallel to the applied electric field. Likewise, when all the inclusions are antiparallel to the applied electric field, the value of ε_{eff}' tends towards Wiener's lower limit while μ_{eff}' tends towards the upper limit.

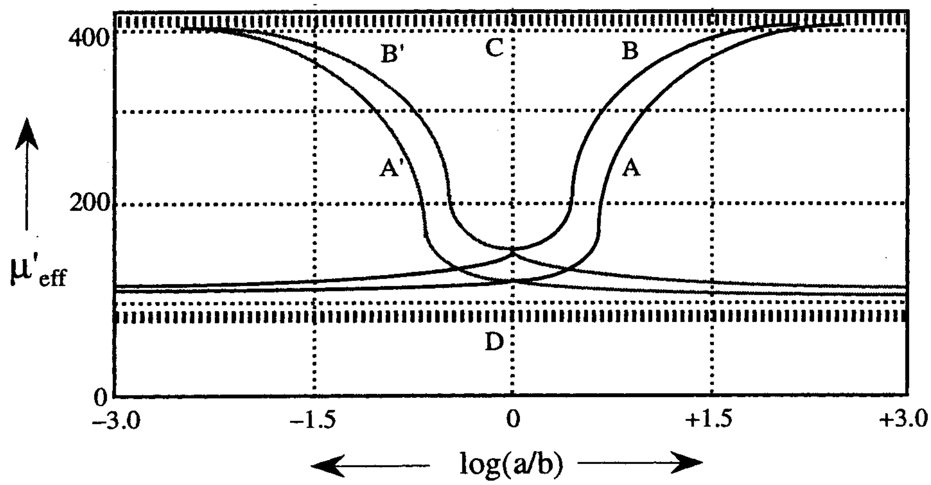

Figure 25.8 Effective permeability (μ_{eff}') of an orderly-textured chiralic mixture *versus* the (a/b) ratio of the inclusions of volume fraction, $\theta = 0.4$.
(Data: $\varepsilon_r = 78.3$; $\mu_r = 1000$; $\varepsilon_2 = 2$; $\mu_2 = 55$; $\gamma = +1$; $\zeta = 0.0001$.)
AA': Sihvola and Lindell's [10] formulations for perpendicular orientation (A) or parallel (A') to the applied electric field (**E**); BB': Corresponding results with the formulations of Equations 25.16 and 25.17; C: Wiener's upper limit; D: Wiener's lower limit.

- Again as before, the above are reversed for oblate spheroidal inclusions.
- Unlike Taylor's formulations discussed in Chapter 4 of ε_{eff} at $e = 0$, Sihvola and Lindell's formulation underestimates the values of both ε_{eff}' and μ_{eff}' at $e = 0$ when compared to the logarithmic law. This is could again be reasoned as due to the dilute phase approximation of Maxwell-Garnett theory on the basis of which Sihvola and Lindell's formulations were derived.
- Use of Langevin's theory enables the construction of the ordered texture from the disordered dispersion regardless of particulate concentration and it also implicitly accounts for the interparticulate interaction within the macroscopic test mixture.

25.10 Applications of Electromagnetic Chiralic Materials

Chirosorb™: A novel material which is invisible to incident to EM waves (by virtue of its zero reflectance characteristics) has been synthesized [14] by embedding randomly oriented identical microstructures (such as microhelices) in an isotropic host medium. This material could be used for RCS reduction purposes.

Chirowaveguide materials: When a waveguide is filled with an appropriate electromagnetic chiralic material, the propagating TE and TM modes become coupled with a coefficient proportional to the chiralic admittance parameter. Possible use of this principle in polymer waveguides and integrated optical devices is indicted in [13].

Chiralic substrate materials: For microstrip antennas use of a chiralic material as a substrate has been suggested. Since chiral materials are polarization sensitive, use of these materials in such structures would enable controlling the antenna beam characteristics, bandwidth, and radiation efficiency.

Chiral-coated EM shieldings: Chiral-coated surfaces are feasible as EM shields compatible for common EM ambients and pulsed electromagnetic (EMP) environments. Such surfaces can also be synthesized for RCS control [24]. *Salisbury shield* realization with chiral substances is also a conceivable product [25].

Chiralic material for EM focusing: Named as *"chirolens"* of spherical geometry, Enghetta and Kowartz have indicated [24] a novel method of realizing two focal points corresponding to two eigen modes of the EM field components present in the chiralic lens material. It is indicated that such lens can focus one of the modes and defocus the other. Potential use of a chirolens as couplers for waveguides, polarization filters, etc. has been suggested.

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Defining Terms

Chirality: (Right or left)-handedness.

Chiralic composite: A composite medium in which an achiralic host medium forms a receptable for the dispersion of chiralic inclusions.

Chiralic materials: Medium wherein electric and magnetic fields are cross-coupled.

Optical rotary dispersion (ORD): Geometry-induced rotation of plane of polarization of a plane wave propagating across a medium.