CHAPTER 11

Superconducting Materials

11.1 Introduction

The phenomenon of *superconductivity* came to be known in 1911 by observing that a capillary column of mercury immersed in a liquid helium bath would show an abrupt reduction in resistance, and at 4.2 K it became impossible to measure the low resistance exhibited by the column with the then available measurement techniques. It was concluded that below a *critical temperature* (T_c) the mercury had passed into a new state which on account of its extraordinary electrical properties was called the *superconducting state.* Subsequent studies on superconduction were pursued with tin and lead as candidate materials and it was discovered that a *critical current density* (J_c) is carried by a superconduction sample before it returns to its normal (metallic) state. Similar to the threshold value of current or critical current density, it was also observed that a *critical magnetic field* (H_c) intensity is required to destroy the superconducting effect in a sample; and the following empirical law was established relating to H_c and T_c:

$$H_c(T) \cong H_{co}[1 - (T/T_c)^2] \tag{11.1}$$

where H_{co} refers to the critical magnetic field intensity at zero temperature and T depicts the operating temperature. Hence superconductivity ceases to exist in a sample with the prevalence of the following influences in excess of certain threshold (critical) values:

- Temperature (T)
- Magnetic field ($\boldsymbol{B}$ or $\boldsymbol{H}$)
- Current density ($\boldsymbol{J}$)

The state of a superconductor can be depicted in terms of $\boldsymbol{B}$ and T as shown in Figure 11.1.

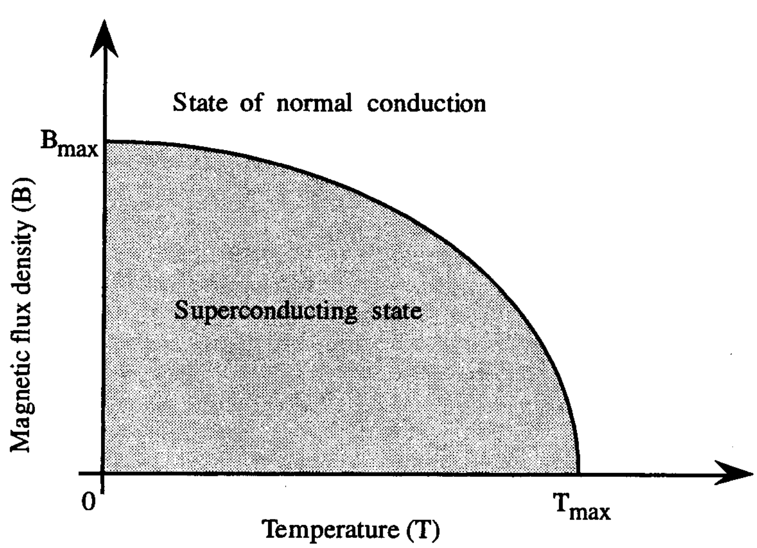

Figure 11.1 Superconducting state as decided by temperature and magnetic flux.

The maximum values of the transitional parameters B_{max} ($= \mu H_{max}$, μ: Permeability of the material) and T_{max} are approximately specified by the relation:

$$B_{max} \cong \alpha T_{max} \tag{11.2}$$

where α is a coefficient equal to 0.02 tesla/K.

A superconductor (unlike the conventional *perfect conductor*) does not *conserve* the magnetic field flux within it. It rather *expels* the flux (assuming that the applied magnetic field is insufficient to destroy the superconduction). A superconductor which seeks to maintain the conduction of magnetic flux density ***(B)*** equal to zero within itself is known as a *perfect diamagnetic*. The phenomenon of expelling the magnetic field by a superconductor is referred to as the *Meissener effect*. Therefore, it has become known that superconductivity is "more than just perfect conductivity".

It has been established that apart from certain pure elements, a combination of superconductive metals or a superconductive metal plus a nonsuperconductive material may also exhibit superconductivity. Similarly there are compounds whose constituent elements themselves may not be superconductors, but the compounds are superconductors. For example, neither Co nor Si nor O is a superconductor, but $CoSiO_2$ exhibits superconductivity with T_c = 1.4 K. It is also to be noted that the well-known good conductors at ordinary temperature such as Cu, Au, Ag, and Pt have not yet been transferred into a superconducting state even at the lowermost cryogenic temperatures feasible in modern technology.

Table 11.1 Typical Superconductors and Their Critical Temperatures

Elements	T_c K	Compounds	T_c K
Tungsten (W)	0.01	AgF_2	0.07
Iridium (Ir)	0.14	$CuAl_2$	1.00
Zinc (Zn)	0.90	$ReYO_3$	2.00
Aluminum (Al)	1.20	NiBi	4.30
Thallium (Tl)	2.40	La_3S_4	8.30
Indium (In)	3.40	$MoRe_3$	10.00
Tin (Sn)	3.70	MoN	12.00
Mercury (Hg)	4.15	MoC	13.00
Tantulum (Ta)	4.50	V_3Ga	14.00
Vanadium (V)	5.30	V_3In & Nb_3Ga	15.00
Lead (Pb)	7.20	V_3Si & NbN	16.00
Niobium (Nb)	9.40	Nb_3Sn	18.00

Typical superconducting materials and their critical temperatures are listed in Table 11.1. The critical magnetic fields are furnished in Table 11.2.

Table 11.2 Some Superconducting Materials and Their Critical Magnetic Fields

Material	B_c (tesla)
Al	0.0099
Va	0.1370
Ga	0.0051
Ir	0.0020
Cd	0.0030
Nb	0.1944
Sn-α	0.0309
Rh	0.0198
Ti	0.0100
Zn	0.0053
Zr	0.0047

The influence of magnetic field on superconducting properties of materials has led to the classification of superconductors into the following categories:

- *Type I superconductors:* These materials lose superconductivity at exposures even at feeble magnetic fields. Examples are: Pure metals except niobium, vanadium, and technetium. Type I materials do not violate the bulk Meissener effect.
- *Type II superconductors*: These have a higher critical magnetic field than Type I counterparts. Examples are: Superconducting alloys and pure metals like niobium, vanadium and technetium. Doping impurities in these materials improve their critical current densities. The disadvantage of these materials is their inhomogeneity. Say, in a binary alloy constituted by two materials 1 and 2, there is a possibility that the superconduction may vanish in the intermediate range of magnetic field specified by the critical values B_{c1} and B_{c2} corresponding to the constitutive materials 1 and 2, respectively. Type II materials may allow magnetic flux to enter the bulk of their volume.
- *Type III superconductors*: These are similar to Type II except that the aforesaid disadvantages are removed *via* proper compositions. Examples are: $NbSn_2$, NbTi, and NbZrTi.

The resistivity *versus* temperature characteristic of a typical superconductor is illustrated in Figure 11.2 along with the corresponding characteristic of a nonsuperconductor.

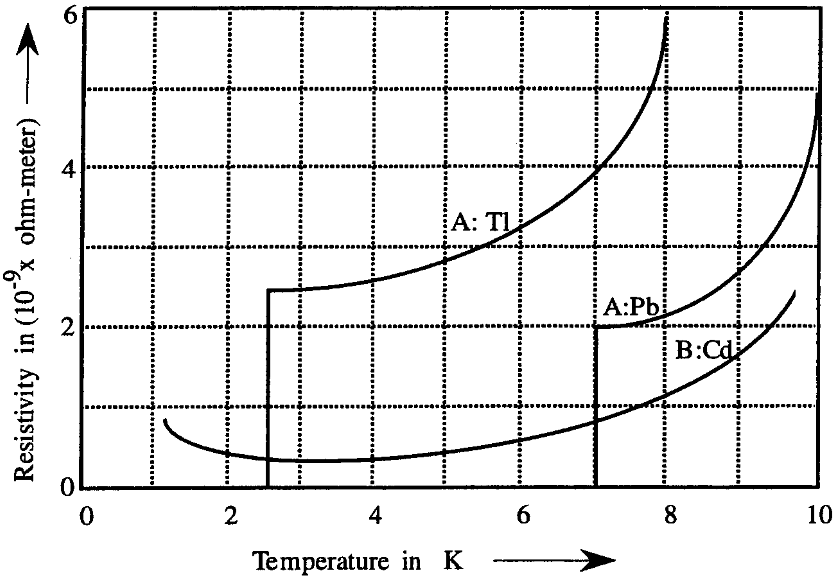

Figure 11.2 Resistivity *versus* temperature.
A: Superconductors;
B: Ordinary conductor which does not pass into a superconducting state.

11.2 Theories and Models of Superconductivity

Classical Model: This model of superconductivity incorporates the concept of zero resistance and perfect diamagnetism into electromagnetic constitutive relations leading to what are known as *London equations*. Relevant expressions due to Fritz and London [1] developed in 1935 are mostly empirical.

This model is based on four fundamental constants which can be associated with the electromagnetic interaction with material or a medium. They are defined as follows:

- *Electromagnetic coupling time* (τ_{em}) delineates what frequencies may be considered low. This *quasistatic regime* is defined by the condition $\omega\tau_{em} << 1$. τ_{em} is equal to $\ell(\mu\varepsilon)^{1/2}$, where ℓ is a geometrical dimension, μ is the permeability, and ε is the permittivity of the material/medium.
- *The charge relaxation time* (τ_e) defines the time over which free charge in the bulk medium relaxes to the surface of the material. It is equal to ε/σ_o where σ_o is the lossy (conductive) attribution to the medium. When $\tau_e > \tau_{em}$, the material/system is considered as *electroquasistatic* corresponding to low frequencies with the energy stored mostly in the electric field.
- *The magnetic diffusion time* (τ_m) is defined as the time required for the magnetic field to distribute itself across a material that is self-consistent with both the applied and induced currents. It is given by $\mu\sigma_o\ell^2$. The *magnetoquasistatic* limit occurs at low frequencies when $\tau_m > \tau_{em}$. Pertinent to superconductors, the magnetoquasistatic limit holds good since it defines the regime of easy conduction of current through the material.
- The average time between successive collisions for a carrier of electric charge in a medium is defined as the *scattering or transport time* (τ_{tr}). For a perfect conductor $\tau_{tr} \rightarrow \infty$.

The interplay between these four *time constants* is considered in defining a *constitutive relation* for a perfect conductor specified by the *first London equation* stated as:

$$E = \partial(\Lambda J)/\partial t \tag{11.3}$$

where Λ is an attributable parameter to a perfectly conducting material and E and J are electric field and current density vectors, respectively. (Note: For a normal conductor with a conductivity σ_o, the corresponding constitutive relation is given by the *microscopic Ohm's law*, namely,: $J = \sigma_o E$.) Similar to the electromagnetic *penetration depth* of a normal conductor, the *characteristic length* (λ_s) parameter at the limiting case of perfect conductivity is given by:

$$\lambda_s = (\Lambda/\mu_o)^{1/2} \tag{11.4}$$

On the basis of the above considerations the classical model of superconductivity incorporates the concept of electrodynamics and thermodynamics to explain the superconductivity phenomenology. Corresponding constitutive field equations are as follows:

$$E = \partial[\bar{\Lambda}(T)\, J_s]/\partial t \qquad \text{(London equation I)} \tag{11.5a}$$

$$-B = \nabla \times [\bar{\Lambda}(T)\, J_s] \qquad \text{(London equation II)} \tag{11.5b}$$

where $\bar{\Lambda}$ is an anisotropic parameter of the superconductor and T is the temperature; and J_s is defined as the current density due to the *superelectrons*. The total current density (J) is therefore:

$$J = J_n + J_s \tag{11.6}$$

where J_n is the current density due to normal electrons.

The above model permits the energy associated with the system being partitioned among electric fields, magnetic fields, and *supercurrents*. The *supercurrent* here refers implicitly to the kinetic energy of the superelectrons. The classical model of superconductivity relies on explaining the relevant superconducting properties on the basis of the aforesaid considerations.

Macroscopic Quantum Model: This *MQM model* was developed to demonstrate that superconduction is a manifestation of quantum mechanical phenomenology. This model enclaves the concept of classical model as well as describes self-consistently the various properties of superconductors. This model also explains the anomaly such as why Type II superconductors violate the Meissener effect. The MQM model also explains the *Josephson junction* prevailing in *small-scale superconducting* systems. (Details on the Josephson junction are presented later.)

The MQM theory is based on the unified aspects of electromagnetism, quantum mechanical considerations of superconductivity, the *Ginzburg-Landau theory* and principles of thermodynamics.

In quantum mechanics, the wave-particle duality of nature is explicit. As a consequence, the so-called *Schrödinger wave equation* for a single quantum particle with a scalar potential describes the dynamical evolution of a probability amplitude (wave) function. The physical attribution of this *wave function*, $\Psi(r,t)$, is that the square of its magnitude is the probability that the particle will be at a specific place r at a certain time t.

Inasmuch as superconductivity can be envisioned as a coherent phenomenon between all the superelectrons, the entire ensemble of such carriers can be represented by a single macroscopic wave function, namely:

$$\Psi(r,t) = [n^*(r,t)]^{1/2}\{\exp[j\theta(r,t)]\} \qquad (11.7)$$

The above equation is akin to the *Schrödinger wave equation* for a single particle. In short, it is possible to assign a single wave function to depict the entire ensemble of carriers subjected to an electromagnetic excitation in a superconductor wherein the local density of the superelectrons in space and time is specified by $n^*(r,t)$. Equation 11.7 governs the probability of current function in the superconductor. That is, inasmuch as Ψ in Equation 11.7 refers to an ensemble of many particles, the proliferation of probability for the entire ensemble is equivalent to the flow of the macroscopic supercurrent J_s; and for an isotropic superconduction the following equation holds good:

$$\Lambda J_S = -[A(r,t) - \hbar\,\nabla\theta(r,t)/q^*] \qquad (11.8)$$

where A is the *magnetic vector potential*, θ is a real function representing the phase of the complex number in Equation 11.7, $\hbar = h/2\pi$ (h being Planck's constant) and q^* is the charge associated with superelectrons.

Equation 11.8 is known as the *supercurrent equation* and it is of primary importance in the macroscopic quantum model of superconduction. The time derivative of Equation 11.8 yields:

$$\partial(\Lambda J)/\partial t = \boldsymbol{E} = \nabla(\Lambda J_S^{\,2}/2)/n^*q^* \qquad (11.9)$$

which is the same as the first London equation, which self-consistently includes the effect of magnetic field created by the motion of supercarriers. The curl of supercurrent equation leads to the *second London equation*, namely:

$$\nabla\times(\nabla J_S) = -\boldsymbol{B} \qquad (11.10)$$

The supercurrent continuity around a closed path is specified by the relation:

$$\oint_C (\Lambda J_S)\cdot d\ell + \iint_S \boldsymbol{B}\cdot ds = n\Phi_O \qquad (11.11)$$

which is referred to as the statement of *fluxoid quantization* where Φ_o is the *flux quantum*. Measurement of Φ_o implicitly confirms that the so-called *Cooper pair* of electrons (as will be described in the next section) identically represents the concept of superelectron. The macroscopic quantum model also applies to superconductors which are anisotropic.

Bardeen-Cooper-Schreiffer model: Known as *BCS theory*, it was developed to explain the microscopic aspects of how superconductivity occurs. The central theme of BCS theory is that the electrons that carry lossy currents in the normal (metallic) state pair together in the superconducting state. Such pairs are referred to as *Cooper pairs* or *superelectrons* signifying the lossless supercurrent they carry. The charge of superelectron is equal to twice the electronic charge, as also is the mass of the superelectrons.

The separation of paired electrons under the influence of temperature and/or magnetic field above a certain critical value would destroy the superconducting property of the material. Further, the paired electrons liberate energy in small doses so that the usual *Joule losses* of power observed in metals with normal conductivity do not occur in superconductors.

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The BCS theory introduces an energy scale on the basis of the bound energy pertinent to a Cooper pair. This bound energy (2Δ) is typically on the order of 10^{-3} eV for conventional superconductors with $T_c \leq 25$ K. This bound energy is called the *energy gap* of the superconductor. It specifies the minimum amount of energy to split the Cooper pair into two unbound electrons.

The BCS theory is not, however, adequate to explain the high temperature superconductivity.

The evolution of BCS theory is as follows: The macroscopic quantum model discussed earlier can be extended to address two other theories, namely, (i) *Ginzburg-Landau* (GL) theory and (ii) the BCS theory. The GL theory, like the macroscopic quantum model is phenomenological in nature. It specifies two fundamental length scales: The *coherence length* ζ and the *penetration depth* λ_s. It is based on writing the *Gibbs free energy* of the superconductor in terms of an *order parameter* $f(r)$ leading to the following two equations:

$$\zeta^2(\Delta/j + 2\pi A^2/\Phi_o)^2 f + |f^2| f - f = 0 \tag{11.12}$$

(GL equation)

and,

$$J_s = (\Phi_o/2\pi\mu_o\lambda_s^2)\ Re[f^*(\Delta/j + 2\pi A/\Phi_o)f] \tag{11.13}$$

(Supercurrent equation)

The above two equations are coupled and refer to the minimization of Gibbs energy under equilibrium.

The *Ginzburg-London* (GL) theory is governed essentially by the aforesaid two characteristic lengths, namely, the coherence length (ξ) and the penetration length (λ_s). The coherence length decides the spatial change in the order parameter whereas the penetration length governs the spatial change in the electromagnetic fields and currents. The coupling of these two lengths would lead to (i) the so-called *Josephson behavior* of the current density. That is, when the order parameter is restricted on a length scale smaller than ξ, there exists a probability that a Cooper pair (or its corresponding macroscopic wave function) may *tunnel* from one superconductor to the other as an ordered, coherent process. (ii) The coupling of the energies due to the magnetic field penetration and variations in the order parameter may also cause the observed differences between Type I and Type II superconductors. The interaction between the Gibbs free-energy H_c and the characteristic lengths ξ and λ_s are:

$$H_c = \Phi_o/(2\sqrt{2}\ \pi\mu_o\lambda_s\xi) \tag{11.14}$$

where μ_o is the free-space permeability and Φ_o is the flux quantum.

Apart from λ_s and ξ, there are two more characteristic lengths *vis-a-vis* superconductors. They are:

- Wavelength of the interacting electromagnetic (EM) field $(2\pi c/\omega)$ where c is the velocity of propagation of the EM wave and $\omega = 2\pi \times$ frequency of the wave
- *Mean free path* (λ_{tr}) which refers to the average distance (in the transport of electrons) between successive collisions or scattering events

In superconductors the two paired electrons scatter in a correlated fashion such that the Cooper pair does not feel the drag force. The absence of such scattering is responsible for the perfect conductivity in a superconductor.

11.3 Applications of Superconductivity

The influence of magnetic field on superconductivity refers to both an external magnetic field or to a magnetic field caused by current passing through the superconductor itself. However, there is a limitation on the current which, without destroying the superconductivity, can be passed through a superconductive circuit. This has constrained the practical use of superconductivity in electrical engineering systems in which high currents have to be passed and strong magnetic fields to be realized. Related studies indicate that an alloy of Nb with 25% of Zr with minimum feasible temperature could lead to a magnetic field density (B_c) limited by the critical value of 10.9 tesla. The corresponding values of B_c for Nb_3Sn is 20 tesla and for $V_{2.95}Ga$ is as high as 35 tesla. These studies indicate the feasibility of producing *cryogenic magnets* with cooled superconducting windings with current densities on the order of 10^9–10^{10} ampere/meter^2. Future successes in this area could lead to systems and devices like electric machines, transformers, etc. operating with virtually at no expense of power.

Another feasible electrical engineering application of superconductors refers to *cryotron* computers, the principle of which is illustrated in Figure 11.3. The central conductor A and the winding B are of two different superconducting materials and are kept at a temperature lower than the critical temperatures of both conductors. The change in current (I_B) in winding B would control the current (I_A) through A; and, if I_B reaches to such a value that the corresponding induced magnetic field destroys the superconductivity of A, the value of I_A will reduce instantaneously posing a switching action (binary transition). The collection of such switches can be designed into a *cryotron computer* with miniaturized film structures.

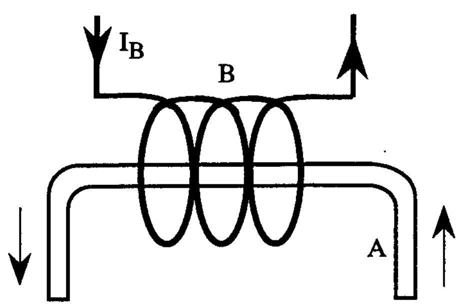

Figure 11.3 Principle of Cryotron computer.

Other uses of superconductors included the following applications:

- Modulators/converters of weak, steady-state currents into audio frequency signals
- Demodulator/rectifier of modulated high-frequency signals making use of the nonlinear behavior of superconductor conductance in the transitional region
- Noncontact commutating switches
- Superconducting memory devices

The following sections describe more specific applications of superconductors.

11.3.1 *Electromagnetic transmission lines*

The transmission lines made of superconducting materials can support a guided wave propagation of EM energy. Such transmission lines can be analyzed *via* electrodynamic principles with appropritate constitutive relations deduced for superconductors beyond the quasistatic approximations. In high frequency, time-varying situations, the parameter Λ that characterizes a superconductor is a function of frequency as well. Pertinent studies include the use of London equations to calculate the EM fields associated with a superconductor and leads to a lumped element model (as illustrated in Figure 11.4) that mimics the behavior of a superconductor.

The feasibility of depicting a superconductor by a conventional set of lumped elements can be extended to represent a transmission line by an electrodynamic structure with the transmission lines being superconductors. Such a model facilitates the elucidation of the associated magnetic field, electric field, normal current density and supercurrent density, components and the *complex propagation constant.* Practical uses of such analysis in the design of waveguide structures and strip lines have been considered and specifically there is a considerable effort directed at microwave transmission line structures of high-T_c and low-T_c superconducting thin films [7,8]. Materials like NbN and $YBa_2Cu_3O_{7-x}$ films have been studied for such applications. Ceramic superconducting wires/transmission lines fabricated from powders of Bi, Sr, Ca, Cu, and O (BSCCO) composition become superconducting above 77 K and sustain magnetic fields above 20 tesla. Such lines are currently in the development stage.

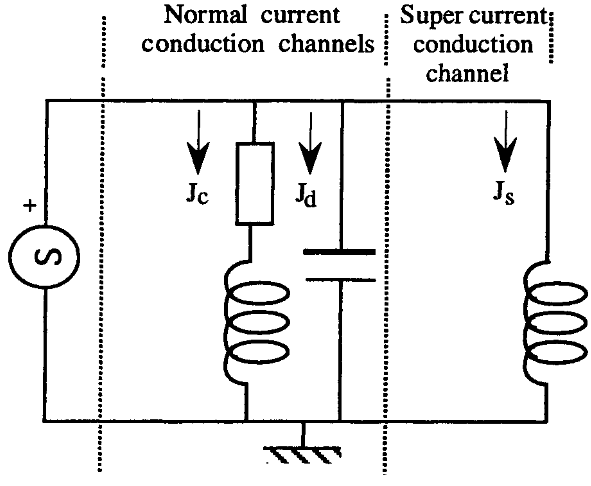

Figure 11.4 A lumped element model of a superconductor.
J_c: Normal conduction current density; J_d: Normal displacement current density;
J_s: Supercurrent density.

11.3.2 *High–T_c superconducting active antennas*

Superconducting materials have also been considered in the development of active antennas. A typical structure refers to using a high-T_c superconducting film (YBCO on MgO substrates) with a corner reflector to detect microwaves.

11.3.3 *Kinetic inductance memory cell*

The Josephson junction refers to a *tunnel junction* between two separated superconductors across which a flow of superconductor can be maintained. Typical current-voltage characteristics of a Josephson junction are illustrated in Figure 11.5.

The current at zero voltage (in Figure 11.5) is a direct result of the Cooper pair tunneling and represents the Josephson current. It flows as a result of Josephson tunneling by the Cooper pair electrons.

When a Josephson junction switches, the voltage across it is typically on the order of millivolts. (On the contrary, conventional semiconductor junctions would require potentional on the order of volts to switch binary states.) Further the superconducting Josephson junction would need three orders of magnitude less power to operate than the standard semiconductor logic devices. Therefore the size of the Josephson memory cells can be extremely small in comparison with semiconductor cells [10].

11.3.4 *Ferroelectric superconductors*

Superconductivity at high temperatures in materials like CuO_2-based perovskite-type structures has indicated a possible relation between superconductivity and ferroelectricity and accordingly relaxor ferroelectric state in CuO-based superconductors (Pb-Bi-Sr-Ca-CuO) has been investigated [11]. Ferroelectric superconductors and viable applications of such materials in practical use provide a niche for upcoming technology.

11.3.5 *Anisotropic superconductors* [1]

In isotropic superconductors, the superconducting properties are not dependent on direction. However, in materials like $NbSe_2$ and $PbMo_6S_2$ and high-T_c superconductors like $Y\,Ba_2Cu_3O_7$, anisotropic behavior is perceived. That is, the conductivity and the parameter Λ are tensors. The use of anisotropic characteristics of superconductors in practical systems rests in futuristic technology trends.

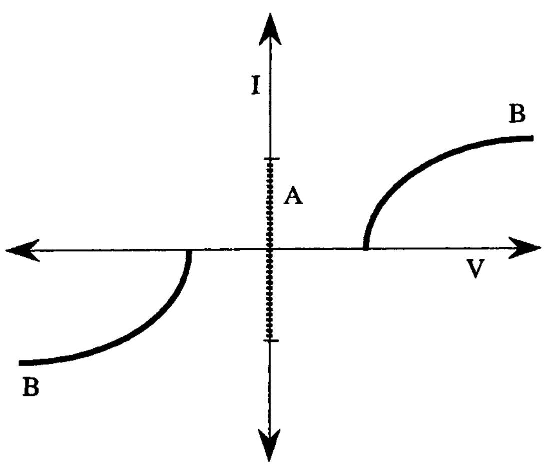

Figure 11.5 Current-voltage characteristics of a Josephson junction. A: Cooper pair tunneling; B: Normal electron tunneling.

11.4 Applications of Superconductors in Electromechanical Systems

The dynamic interaction between a current carrying conductor and a magnetic field is well-known through *Ampere's force law*. In the event of a superconductor being subjected to a magnetic field, an electromechanical *levitation* can be realized. For example, a samarium-cobalt magnet can be levitated by an $YBa_2Cu_3O_7$ superconducting disk, inasmuch as the superconductor (being a perfect diamagnet) prevents the penetration of the magnetic flux through it; and the resulting force of repulsion would levitate the magnet placed in its vicinity.

The concept of magnetic levitation (*maglev*) is being considered for use in levitating a train car above the track to counteract the frictional drag. However, the relevant technology at present is still in its cradle of development.

11.5 The DC SQUID (Superconducting Quantum Interference Device)

SQUID magnetometer is a transducer which produces a voltage signal related to the applied magnetic field. Such SQUIDs make use of superconducting materials and offer extremely high sensitivities [6]. The operation of a SQUID is based on the maximum current through junctions in parallel, being dependent on the magnetic flux enclosed by the loop. Magnetic flux density on the order of 10^{-6} to 10^{-7} weber/meter^2 can be measured with a time constant of 1 sec using SQUIDs. SQUID magnetometers can sense the feeble disturbances in the earth's magnetic field caused nearby submarines, enabling target locations.

11.6 Other Applications

11.6.1 *Radiation detection*

When a superconductor is kept just above its critical temperature (where its resistance varies significantly with temperature), incident radiations can be sensed, inasmuch as such radiations would induce temperature changes (and hence resistance changes) in the superconducting medium.

11.6.2 *Heat valves*

Thermal conductivity of some superconductors would change (increase) by two orders of magnitude when the material is turned into a normal conductor by a magnetic field. This phenomenon can be used to devise a *heat valve* in refrigerating systems operating at cryogenic temperatures.

11.6.3 *Resonant cavities*

The transmission line function of a superconductor can be logically extended to make microwave cavities with very high Q factors, if the cavity walls are coated with a thin film of a superconductor offering low loss characteristics.

11.6.4 *Oxide superconductors*

In an attempt to realize high temperature superconductors, oxide superconductors have emerged in the recent past. Yttrium-barium-copper oxide (YBCO) material has a critical temperature between 90 and 100 K and $Tl_2Ba_2Ca_2Cu_3O_{10}$ (TBCCO) material has yielded $T_c = 125$ K.

11.6.5 *High-field magnets*

In the present, the most important use of superconductors is in producing high magnetic fields. A magnetic flux density of 20 tesla can be produced by superconductor–based solenoids of about 12 × 20 cm. In comparison, even to realize 5 tesla with conventional conductors, the size of the solenoid would be enormous and would need megawatts of power and an exorbitant cooling system.

Type II superconductors are used to produce high magnetic fields since they are superconducting even under large magnetic fields. Typically Nb-Ti alloy and intermetallic compounds such as Nb_3Sn are used in practice.

11.7 Properties of Typical Superconductors

Table 11.3 Properties of Superconductors

Type I Superconductors

Material	T_c (K)	λ_0(nm)	ξ_0(nm)	Δ_0(meV)	H_{co}(mT)
Al	1.18	50	1600	0.18	10.5
In	3.41	65	360	0.54	23.0
Sn	3.72	50	230	0.59	30.5

Material	T_c (K)	λ_o(nm)	ξ_o(nm)	Δ_o(meV)	H_{co}(mT)
Pb	7.20	40	90	1.35	80.0
Nb	9.25	85	40	1.50	198.0

Type II Superconductors: Conventional Types

Material	T_c (K)	$\lambda_{GL}(0)$(nm)	$\xi_{GL}(0)$(nm)	Δ_o(meV)	$H_{c2,o}$(T)
Pb-In	7.0	150	30	1.2	0.2
Pb-Bi	8.3	200	20	1.7	0.5
Nb-Ti	9.5	300	4	1.5	13.0
Nb-N	16.0	200	5	2.4	15.0
$PbMo_6S_8$	15.0	200	2	2.4	60.0
V_3Ga	15.0	90	2-3	2.3	23.0
V_3Si	16.0	60	3	2.3	20.0
Nb_3Sn	18.0	65	3	3.4	23.0
Nb_3Ge	23.0	90	3	3.7	38.0

Type III Superconductors: High-Temperature Versions

Material	T_c (K)	$\lambda_{a,b}$(nm)	λ_c(nm)	$\xi_{a,b}$(nm)	ξ_c(nm)
$La_{1.85}Sr_{0.5}CuO_4$	40	80	400	4	0.7
$YBa_2Cu_3O_7$	95	30	200	3	0.4
$Bi_2Sr_2CaCu_2O_8$	85	25	500	4	0.2
$Bi_2Sr_2Ca_2Cu_3O_{10}$	110				
$Tl_2Ba_2CaCu_2O_8$	108				
$Tl_2Ba_2Ca_2Cu_3O_{10}$	125				

Type I and II with permission from R.J. Donnelly, "Cryogenics" in Physics Vade Mecum, H.L. Anderson (Ed.) American Institute of Physics: 1981; Type III with permission from T.P. Orlando and K.A. Duelin, Foundations of Applied Superconductivity, Addison-Wesley Publishing Co.: 1991.

Representative values of the parameter of typical superconductors are furnished in Table 11.3.

The values in the above tables are for clean elements. The penetration depth λ_o is given at zero temperature, as are the coherence length ξ_o, the thermodynamic critical field H_{co}, and the energy gap Δ_o.

Further, the values are only representative because the parameter for alloys and compounds depends on how clean or dirty the material is. The penetration depth $\lambda_{GL}(o)$ is given as the coefficient of the Ginzburg-Landau temperature dependence as $\lambda_{GL}(T) = \lambda_{GL}(0)(1 - T/T_o)^{-1/2}$ and likewise for the coherence length where $\xi_{GL}(T) = \xi_{GL}(0)(1 - T/T_c)^{-1/2}$. The upper critical field $H_{c2,o}$ is given at zero temperature as well as the energy gap Δ_o.

The values in Table 11.3 are only approximate because the parameters for high-temperature superconductors have not all been established well enough. The penetration depth is the coefficient of the Ginzburg-Landau temperature dependence $\lambda_{GL}(0)$ as in the table for conventional superconductors; likewise for the coherence length, $\xi_{GL}(0)$. However, since these materials are anisotropic, these lengths are given along the principal axis. The directions $\hat{a}$ and $\hat{b}$ are taken to lie in the plane of the Cu-O planes and $\hat{c}$ is taken to be perpendicular to that plane.

11.8 Concluding Remarks

Superconductors are the most intriguing materials of modern times. Realizing a high temperature superconductor with potentials for technological applications is the target of scientific studies around the world. Though some breakthroughs have been achieved, a comprehensive set of materials for wide-scale applications is yet to be conceived.

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Defining Terms

BCS theory: A theory that suggests the superconduction being instigated by a pair of electrons known as superelectrons.

Cooper pairs: Pair of superelectrons responsible for superconductivity as per BCS theory.

Josephson behavior: The behavior of current density in a superconductor as controlled by coherence length and penetration depth.